SEM-2 (MATHEMATICS GENERAL)

ANUP RAY

Answer (Home work problems)

1) $e^{6y}\left(\frac{1}{2}x^2y^2 - \frac{1}{3}x^3 + \frac{1}{6}y^2 - \frac{1}{18}y + \frac{1}{108}\right) = c$ 3) $x^3y^3 + x^2 = cy$

2) $3x^2y^4 + 6xy^2 + 2y = c$ 4) $x(y + 2/y^2) + y^2 = c$

Rule 4: If the given equation $Mdx + Ndy = 0$ is homogeneous and $Mx + Ny \neq 0$, then $1/(Mx+Ny)$ is an integrating factor.

Ex: Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

Ans: we have $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ --- (1)

clearly (1) is a homogeneous differential Equation

compairing (1) with the equation $Mdx + Ndy = 0$, we get

$$M = x^2y - 2xy^2, \quad N = -(x^3 - 3x^2y)$$

$$\therefore Mx + Ny = x^3y - 2x^2y^2 - x^3y + 3x^2y^2 = x^2y^2 \neq 0$$

$$\therefore \text{I. F. of (1)} = \frac{1}{Mx+Ny} = \frac{1}{x^2y^2},$$

Multiplying both sides of (1) by $\frac{1}{x^2y^2}$ (ie I.F.) we get,

$$\left(\frac{1}{y} - \frac{2}{x}\right)dx - \left(\frac{x}{y^2} - \frac{3}{y}\right)dy = 0,$$ which is must exact.

$$\therefore \int M dx \text{ (Taking } y \text{ as constant)} = \int\left(\frac{1}{y} - \frac{2}{x}\right)dx$$

$$= \frac{x}{y} - 2\log x.$$

Also $\int N dy$ (Terms free from x) $= \int -\frac{3}{y}\,dy = -3\log y$

Hence the general solution is

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$\frac{x}{y} - 2\log x + 3\log y = \log c$, where c is an arbitrary integration constant.

or $\log y^3 - \log x^2 - \log c = -x/y$

or $y^3 = cx^2 e^{-x/y}$

Home work

1) Is it possible to solve the following equations in Rule-4? If so, solve it completely:

a) $(x^2+y^2)\,dx - 2xy\,dy = 0$

b) $(x^2+y^2)\,dx + x(x-2y)\,dy = 0$

2) Solve $(x^3+y^3)\,dx - xy^2\,dy = 0$

3) Solve $(x+y)\,dy + (y-x)\,dx = 0$

4) Show that $(x+y+1)^{-4}$ is an integrating factor of
$(2xy - y^2 - x)\,dx + (2xy - x^2 - x)\,dy = 0$

5) Show that the equation $(x^3 - 3x^2y + 2xy^2)\,dx - (x^3 - 2x^2y + y^3)\,dy = 0$ is exact and ~~the~~ find the solution if $y=1$, when $x=1$.